System: $y(t) = x(t) * h(t)$

For continuous time $f(t^-) = f(t^+)$

For discrete time f[n] may not be equal to f[n+1] pr f]n-1]

Energy & Power:

$$E_c = \int_{t_1}^{t_2} |x(t)|^2 \, dt \longrightarrow P_c = \frac{E_c}{t_2 - t_1}$$

$$E_d = \sum_{n=-N}^{N} |x[n]|^2 \longrightarrow P_d = \frac{E_d}{2N+1}$$

$$E_\infty = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt \qquad P_\infty = \frac{E_\infty}{2T}$$

$$|x[n]| = \sqrt{x[n]x^*[n]} \qquad P_\infty = \frac{E_\infty}{2N+1}$$

Periocity of Signals:

$x(t) = x(t+T)$ for continous time

$x[n] = x[n+N]$ for discrete time

$e^{j2\pi m} = 1$

Properties of Impulse Function:

1) $\delta(t-t_0)f(t) = \delta(t-t_0)f(t_0) = f(t_0)$

2) $\delta(t-t_0)f(t-t_1) = \delta(t-t_0)f(t_0-t_1)$

3) $\int_{-\infty}^{\infty} \delta(t)dt = 1$ OR $\int_{-\infty}^{\infty} \delta(t-t_0)dt = 1$

4) $\int_{-\infty}^{\infty} \delta(t-t_0)f(t)dt = f(t_0)$,

$$\int_{-\infty}^{\infty} \delta(t-t_0)f(t-t_1)dt = f(t_0-t_1)$$

Ramp Function:

$$u(t) = \frac{dr(t)}{dt}, r(t) = \int_{-\infty}^{t} u(t)dt$$

$$\delta(t) = \frac{du(t)}{dt}, u(t) = \int_{0}^{\infty} \delta(t)dt$$

$$\delta(t) = \frac{d^2 r(t)}{dt^2} \qquad \delta(at) = \frac{1}{|a|}\delta(t)$$

$$r(t) \underset{\text{integration}}{\overset{\text{derivative}}{\rightleftarrows}} u(t) \underset{\text{integration}}{\overset{\text{derivative}}{\rightleftarrows}} \delta(t)$$

Even and Odd Signals:

Even $\rightarrow x(t) = x(-t)$, Odd $\rightarrow x(t) = -x(t)$

Exponential and Sinusoidal Signals:

$$e^{jw_0 t} = \cos(w_0 t) + j\sin(w_0 t) \qquad w_0 = 2\pi f$$

Discrete Time Signal, Base Functions:

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

$$r[n] = \sum_{k=0}^{\infty} u[n-k] = \sum_{k=0}^{\infty} k\delta[n-k]$$

Linearity:

$$ax_1(t) + bx_2(t) = ay_1(t) + by_2(t)$$

Linear Time Invariant Systems:

The Convolution Sum:

$$g[n] = \sum_{k=-\infty}^{\infty} x[k] \times h[n-k]$$

$$y[n] = \sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha}$$

Properties of Convolution:

Commutative Property:

$$x[n]*h[n] = h[n]*x[n]$$

Distributive Property:

$$x[n]*(h_1[n] + h_2[n]) = x[n]*h_1[n] + x[n]*h_2[n]$$

Associative Property:

$$x(t)*[h_1(t)*h_2(t)] = (x(t)*h_1(t))*h_2(t)$$

Invertability of LTI Systems:

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

Stability for LTI Systems:

LTI system is stable if and only if

$$\int_{-\infty}^{\infty} | h(\tau) | d\tau < \infty$$

Unit Step Response:

For discrete time

$$s[n] = u[n] * h[n]$$

$$s[n] = \sum_{k=-\infty}^{\infty} h[k]u[n-k] \qquad n \geq k$$

Fourier Analysis for Continuous Time Signals and Systems :

$$x(t) = A[0] + \sum_{k=1}^{\infty} A[k]\cos(\frac{2\pi}{T}kt) + \sum_{k=1}^{\infty} B[k]\sin(\frac{2\pi}{T}kt)$$

$$A[0] = \frac{1}{T}\int_T x(t)dt$$

$$A[k] = \frac{2}{T}\int_T x(t)\cos(k\frac{2\pi}{T}t)dt$$

$$B[k] = \frac{2}{T}\int_T x(t)\sin(k\frac{2\pi}{T}t)dt$$

Fourier series representation

$$x(t) = \sum_{k=-\infty}^{\infty} x[k]e^{jk\frac{2\pi}{T}t}$$

$$x[k] = \frac{1}{T}\int_T x(t)e^{-jk\frac{2\pi}{T}t}dt$$

$$A[k] = (x[k] + x*[k])x2 \qquad k \neq 0$$

$$B[k] = (x[k] - x*[k]j)$$

$A[k] = x[0]$ OR $x[k] = \frac{1}{2}(A[k] - jB[k])$

$$A[0] = x[0]$$

$y(t) = \frac{d^n x(t)}{dt^n}$ then $y[k] = (jw_0 k)^n \times x[k]$

$$y(t) = \frac{d^n x(t)}{d^n t} \qquad x[k] = \left(\frac{1}{jkw_0}\right)^n y[k]$$

$$\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$

$$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

Properties of Fourier Series Coefficients:

Signal		FSC
1. $Ax(t) + By(t)$	$\xleftrightarrow{FSC}$	$Ax[k] + By[k]$
2. $x(t - t_0)$	$\xleftrightarrow{FSC}$	$x[k]e^{-jk\frac{2\pi}{T}t_0}$
3. $e^{jm\frac{2\pi}{T}t}x(t)$	$\xleftrightarrow{FSC}$	$x[k-m]$
4. $x*(t)$	$\xleftrightarrow{FSC}$	$x*[-k]$
5. $x(-t)$	$\xleftrightarrow{FSC}$	$x[-k]$
6. $\int_T x(\tau)y(t-\tau)d\tau$	$\xleftrightarrow{FSC}$	$Tx[k]y[k]$

7.

$$x(t)y(t) \xleftrightarrow{FSC} x[k] * y[k] = \sum_{k=-\infty}^{\infty} x[l]y[k-l]$$

8. $\frac{d^n x[n]}{dt^n} \xleftrightarrow{FSC} (jkw_0)^n x[k]$

9. $\int_{-\infty}^{t} x(t)dt \xleftrightarrow{FSC} \frac{1}{jkw_0}x[k]$

10. $\int_T | x(t) |^2 \, dt \xleftrightarrow{FSC} T\sum_{k=-\infty}^{\infty} | x[k] |^2$

Representation of a periodic Signals:

The continuous time fourier transform:

$$\widetilde{x(t)} = \sum_{k=-\infty}^{\infty} x(t-kT)$$

$x(w) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$ (Fourier transform of a Periodic Signal x(t))

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(w)e^{jwt}dw$ (Inverse Fourier Transform of a Periodic Signal)

Convergence of Fourier Transform:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$|e^{j\theta}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

Relation between FSC of Periodic Signal:

$$\widetilde{x[k]} = \frac{1}{T} x(w)|_{w=kw_0}, \quad w_0 = \frac{2\pi}{T}$$

$$e^{jw_0t} \xleftrightarrow{F.T} 2\pi\delta(w - w_0)$$

Properties of Fourier Transform:

$$|x(w)| = \sqrt{a^2(w) + b^2(w)}$$

$$\tan^{-1} = \frac{b(w)}{a(w)}$$

$$\frac{d^n x(t)}{dt^n} \xleftrightarrow{FT} (jw)^n x(w)$$

Fourier Analysis for Discrete Time Signals and Systems:

$$x[n] = \sum_{\infty}^{N-1} x[k] e^{jk\frac{2\pi}{N}n}$$

$$x[k] = \frac{1}{N}\sum_{n,N} x[n] e^{-jk\frac{2\pi}{N}n}$$

$$\sum_{n=0}^{N-1} r^n = 1 + r + r^2 + r^3 + \cdots = \frac{1-r^n}{1-r} \quad \text{if } r \neq 1$$

$$\sum_{n=0}^{N-1} r^n = 1 + r + r^2 + r^3 + \cdots = N \quad \text{if } r = 1$$

Properties of Discrete Fourier Series Coefficients:

1. $\alpha x[n] + \beta y[n] \xleftrightarrow{DFSC} \alpha x[k] + \beta y[k]$

2. $x[n - n_0] \xleftrightarrow{DFSC} e^{-jk\frac{2\pi}{N}n_0} x[k]$

3. $e^{jm\frac{2\pi}{N}n} x[n] \xleftrightarrow{DFSC} x[k-m]$

4. $x^*[n] \xleftrightarrow{DFSC} x^*[k]$

5. $x[-n] \xleftrightarrow{DFSC} x[-k]$

6. $\sum_{m,n} x[m]\, y[n-m] \xleftrightarrow{DFSC} Nx[k]\, y[k]$

7. $x[n]y[n] \xleftrightarrow{DFSC} \sum_{l,N} x[l]y[k-l]$

8. $x[n] - x[n-l] \xleftrightarrow{DFSC} (1 - e^{-jk\frac{2\pi}{N}})x[k]$

9.

$$y[n] = \begin{cases} x[n/m] & \text{if } n \text{ is multiple of } m \\ 0 & \text{else} \end{cases} \xleftrightarrow{DFSC} y[k] = \frac{1}{m} x[k]$$

10. $\sum_{k=-\infty}^{n} x[k] \xleftrightarrow{DFSC} \frac{x[k]}{1 - e^{-jk\frac{2\pi}{N}}}$

The Laplace Transform:

$$x(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \quad = \quad FT\left(x(t)e^{-\theta t}\right)$$

Properties of Laplace Transform:

$$x_1(t) \xleftrightarrow{L} x_1(s) \qquad ROC = R_1$$

$$x_2(t) \xleftrightarrow{L} x_2(s) \qquad ROC = R_2$$

$$x(t) \xleftrightarrow{L} x(s) \qquad ROC = R$$

1)Linearity

$$ax_1(t) + bx_2(t) \xleftrightarrow{L} ax_1(s) + bx_2(s)$$

ROC $R_1 \cap R_2$

2)Time shifting $x(t) \xleftrightarrow{L} x(s) \quad ROC = R$

$x(t - t_0) \xleftrightarrow{L} e^{-st_0} x(s)$ with $ROC = R$

3)Shifting in the S-Domain

$x(t) \xleftrightarrow{L} x(s)$, $ROC = R$

$$e^{s_0 t}x(t)\overset{L}{\longleftrightarrow}x(s-s_0)$$
$$ROC = R+\text{Re}\{s_0\}$$

4)Time Scaling

$$x(at)\overset{L}{\longleftrightarrow}\frac{1}{|a|}x\left(\frac{s}{a}\right),\quad ROC=\frac{R}{a}$$

5)Convolution Property

$$x_1(t)*x_2(t)\overset{L}{\longleftrightarrow}x_1(s)x_2(s)$$
$$ROC = R_1 \cap R_2$$

6)Differentation in Time Domain

$$\frac{dx(t)}{dt}\overset{L}{\longleftrightarrow}sx(s)\quad ROC > R$$

7)Differentation in s-domain

$$-tx(t)\overset{L}{\longleftrightarrow}\frac{dx(s)}{ds}\quad ROC = R$$

8)Integration in the time domain

$$\int_{-\infty}^{t}x(\tau)\,d\tau\overset{L}{\longleftrightarrow}\frac{1}{s}x(s)$$
$$ROC > R\cap\{\text{Re}(s)>0\}$$

9)The initial and final value theorems

$$x(0^+)=\lim_{s\to\infty}sx(s)\qquad \lim_{t\to\infty}x(t)=\lim_{s\to 0}sx(s)$$

<u>The Unilateral Laplace Transform:</u>

$$\frac{dx(t)}{dt}\overset{L}{\longleftrightarrow}sx(s)-x(0)$$

Second derivative of x(t):

$$L\{x''(t)\}=s^2x(s)-sx(0)-x'(0)$$

<u>The Z-Transform:</u>

$$x(z)=\sum_{n=-\infty}^{\infty}x[n]z^{-n}\ ,x(z)=FT\{x[n]r^{-n}\}$$

Note:

$$\sum_{n=0}^{\infty}r^n=\frac{1}{1-r},\quad |r|<1$$

<u>Properties of Z-Transform</u>

$$x_1[n]\overset{z}{\longleftrightarrow}x_1[z]\quad ROC_1$$

$$x_2[n]\overset{z}{\longleftrightarrow}x_2[z]\quad ROC_2$$

1)Linearity

$$\alpha x_1[n]+\beta x_2[n]\overset{z}{\longleftrightarrow}\alpha x_1[z]+\beta x_2[z]$$
$$ROC = ROC_1\cap ROC_2$$

2)Time Shifting

$$x[n]\overset{z}{\longleftrightarrow}x[z]\quad ROC = R_x$$

$$x[n-n_0]\overset{z}{\longleftrightarrow}z^{-n}x[z]\quad ROC = R_x$$

3)Frequency Shifting

$$e^{jn_0}x[n]\overset{z}{\longleftrightarrow}x(e^{-jn_0}z)\quad ROC = R_x$$

$$z_0{}^n x[n]\overset{z}{\longleftrightarrow}x\left(\frac{z}{z_0}\right)\quad ROC = z_0R_x$$

4)Time Reversal

$$x[-n]\overset{z}{\longleftrightarrow}x\left(\frac{1}{z}\right),\quad ROC=\frac{1}{R_x}$$

5)Convolution Property

$$x_1[n]*x_2[n]\overset{z}{\longleftrightarrow}x_1(z)x_2(z)$$
$$ROC = ROC_1\cap ROC_2$$

6)Differantiation in the z-domain

$$nx[n]\overset{z}{\longleftrightarrow}-\frac{zdx(z)}{dz},\quad ROC = R_x$$

7)The initial value theorem

If $x[n]=0$, $n<0$ then $x[0]=\lim_{z\to\infty}x(z)$

Table of z-Transform Pairs

	$x[n] = \mathcal{Z}^{-1}\{X(z)\} = \frac{1}{2\pi j}\oint X(z)z^{n-1}dz$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$	*ROC*
transform	$x[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$X(z)$	R_x
time reversal	$x[-n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$X(\frac{1}{z})$	$\frac{1}{R_x}$
complex conjugation	$x^*[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$X^*(z^*)$	R_x
reversed conjugation	$x^*[-n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$X^*(\frac{1}{z^*})$	$\frac{1}{R_x}$
real part	$\mathfrak{Re}\{x[n]\}$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	R_x
imaginary part	$\mathfrak{Im}\{x[n]\}$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	R_x
time shifting	$x[n-n_0]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$z^{-n_0}X(z)$	R_x
scaling in $\mathcal{Z}$	$a^n x[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$X\left(\frac{z}{a}\right)$	$\|a\|R_x$
downsampling by N	$x[Nn], N \in \mathbb{N}_0$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{1}{N}\sum_{k=0}^{N-1} X\left(W_N^k z^{\frac{1}{N}}\right) \quad W_N = e^{-\frac{j2\omega}{N}}$	R_x
linearity	$ax_1[n] + bx_2[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$aX_1(z) + bX_2(z)$	$R_x \cap R_y$
time multiplication	$x_1[n]x_2[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{1}{2\pi j}\oint X_1(u)X_2\left(\frac{z}{u}\right)u^{-1}du$	$R_x \cap R_y$
frequency convolution	$x_1[n] * x_2[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$X_1(z)X_2(t)$	$R_x \cap R_y$
delta function	$\delta[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	1	$\forall\ z$
shifted delta function	$\delta[n-n_0]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	z^{-n_0}	$\forall\ z$
step	$u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{z-1}$	$\|z\| > 1$
	$-u[-n-1]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{z-1}$	$\|z\| < 1$
ramp	$nu[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{(z-1)^2}$	$\|z\| > 1$
	$n^2u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z(z+1)}{(z-1)^3}$	$\|z\| > 1$
	$-n^2u[-n-1]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z(z+1)}{(z-1)^3}$	$\|z\| < 1$
	$n^3u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z(z^2+4z+1)}{(z-1)^4}$	$\|z\| > 1$
	$-n^3u[-n-1]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z(z^2+4z+1)}{(z-1)^4}$	$\|z\| < 1$
	$(-1)^n$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{z+1}$	$\|z\| < 1$
exponential	$a^n u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{z-a}$	$\|z\| > \|a\|$
	$-a^n u[-n-1]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{z-a}$	$\|z\| < \|a\|$
	$a^{n-1}u[n-1]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{1}{z-a}$	$\|z\| > \|a\|$
	$na^n u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{az}{(z-a)^2}$	$\|z\| > \|a\|$
	$n^2a^n u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{az(z+a}{(z-a)^3}$	$\|z\| > \|a\|$
	$e^{-an}u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{z-e^{-a}}$	$\|z\| > \|e^{-a}\|$
exp. interval	$\begin{cases} a^n & n = 0, \ldots, N-1 \\ 0 & \text{otherwise} \end{cases}$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\|z\| > 0$
sine	$\sin(\omega_0 n)\,u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z\sin(\omega_0)}{z^2-2\cos(\omega_0)z+1}$	$\|z\| > 1$
cosine	$\cos(\omega_0 n)\,u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z(z-\cos(\omega_0))}{z^2-2\cos(\omega_0)z+1}$	$\|z\| > 1$
	$a^n\sin(\omega_0 n)\,u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{za\sin(\omega_0)}{z^2-2a\cos(\omega_0)z+a^2}$	$\|z\| > a$
	$a^n\cos(\omega_0 n)\,u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z(z-a\cos(\omega_0))}{z^2-2a\cos(\omega_0)z+a^2}$	$\|z\| > a$
differentiation in $\mathcal{Z}$	$nx[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$-z\frac{dX(z)}{dz}$	R_x
integration in $\mathcal{Z}$	$\frac{x[n]}{n}$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$-\int_0^z \frac{X(z)}{z}dz$	R_x
	$\frac{\prod_{i=1}^{m}(n-i+1)}{a^m m!}a^m u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{(z-a)^{m+1}}$	

Note:

$$\frac{z}{z-1} = \frac{1}{1-z^{-1}}$$

Table of Laplace Transform Pairs

	$f(t)=\mathcal{L}^{-1}\{F(s)\}=\frac{1}{2\pi j}\lim_{T\to\infty}\int_{c-jT}^{c+jT}F(s)e^{st}ds$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F(s)=\mathcal{L}\{f(t)\}=\int_{-\infty}^{+\infty}f(t)e^{-st}dt$	
transform	$f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F(s)$	
complex conjugation	$f^*(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F^*(s^*)$	
time shifting	$f(t-a) \quad t \geqslant a > 0$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$a^{-as}F(s)$	
	$e^{-at}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F(s+a)$	frequency shifting
time scaling	$f(at)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{\lvert a\rvert}F(\frac{s}{a})$	
linearity	$af_1(t)+bf_2(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$aF_1(s)+bF_2(s)$	
time multiplication	$f_1(t)f_2(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F_1(s)*F_2(s)$	frequency convolution
time convolution	$f_1(t)*f_2(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F_1(s)F_2(s)$	frequency product
delta function	$\delta(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	1	
shifted delta function	$\delta(t-a)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	e^{-as}	exponential decay
unit step	$u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}$	
ramp	$tu(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s^2}$	
parabola	$t^2u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2}{s^3}$	
n-th power	t^n	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{n!}{s^{n+1}}$	
exponential decay	e^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s+a}$	
two-sided exponential decay	$e^{-a\lvert t\rvert}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2a}{a^2-s^2}$	
	te^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{(s+a)^2}$	
	$(1-at)e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{(s+a)^2}$	
exponential approach	$1-e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{a}{s(s+a)}$	
sine	$\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2+\omega^2}$	
cosine	$\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2+\omega^2}$	
hyperbolic sine	$\sinh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2-\omega^2}$	
hyperbolic cosine	$\cosh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2-\omega^2}$	
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{(s+a)^2+\omega^2}$	
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s+a}{(s+a)^2+\omega^2}$	
frequency differentiation	$tf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$-F'(s)$	
frequency n-th differentiation	$t^nf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$(-1)^nF^{(n)}(s)$	
time differentiation	$f'(t)=\frac{d}{dt}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$sF(s)-f(0)$	
time 2nd differentiation	$f''(t)=\frac{d^2}{dt^2}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^2F(s)-sf(0)-f'(0)$	
time n-th differentiation	$f^{(n)}(t)=\frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$	
time integration	$\int_0^t f(\tau)d\tau=(u*f)(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}F(s)$	
frequency integration	$\frac{1}{t}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\int_s^\infty F(u)du$	
time inverse	$f^{-1}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)-f^{-1}}{s}$	
time differentiation	$f^{-n}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)}{s^n}+\frac{f^{-1}(0)}{s^n}+\frac{f^{-2}(0)}{s^{n-1}}+\ldots+\frac{f^{-n}(0)}{s}$	